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Project report

**Design optimization of phone casings for sound
vibration damping: preliminary studies on the
Euler–Bernoulli beam model.**

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1 Introduction

In the design of table top phones that contain loudspeakers, for instance conference phones, it is desirable to minimize feedback from the loudspeaker to the microphone. The current work concerns the feedback from the loudspeaker to the microphone through the casing. The final aim is to find a design of the casing that minimizes the structural feedback. A simplified model of the casing, a thin plate, is considered in this preliminary investigation. Laterally uniform deflections of the plate are modelled as the dynamic Euler–Bernoulli beam loaded at the boundary.

The simulation results are verified against 3D structural mechanics solutions from COMSOL 4.2. The thickness distribution of the beam model is numerically optimized in order to minimize the vibrations at a selected region of the beam. A lower limit and an upper limit for beam thickness, as well as the beam weight is used as optimization constraints. A static compliance constraint is also tested. The simulations show that it is possible to move structural vibrations both in the frequency space and in the spatial space. Vibration reduction up to 20 dB of certain frequencies at the microphone position can be obtained.

1.1 Literature study

1.1.1 Euler–Bernoulli beam description

Detailed analyses of Euler–Bernoulli beam frequency response properties are done in several research papers. Most of them considers discontinuous beams, which require a special treatment or modifications to the original beam equation [10]. Yu et al. [11] investigated the transmission spectrum of Euler–Bernoulli beams with locally resonant structures with two degrees of freedom. Both theoretical considerations and experiments were done. Their results indicate a gap in the transmission spectrum for certain frequencies. Koplou et al. [6] considered discontinuous Euler–Bernoulli beams and their frequency response. They derived an improved method for predicting the dynamic response of discontinuous beams and verified the findings experimentally.

1.1.2 Structural acoustics optimization

The vibrations of the dynamic Euler–Bernoulli beam has only rarely been addressed with an optimization formulation. Sorokin et al. [8] [9] optimized the energy transfer in large

systems composed of beam elements. For the description of each element, the dynamic Euler–Bernoulli beam equation was used. An example of such a system is a pipeline. The pipeline terminal point location and properties are used as design variables.

More common for structural acoustics optimization are 2D cases with a structure–environment interaction. Christensen et al. [2] [3] optimized the sound emission directionality of a vibrating structure that is interacting with the surrounding media.

Olhoff et al. [7] applied topology optimization to structures subject to a forced vibration. Their conclusion is that it is possible to move the eigenmode frequencies away from the excitation frequency and thus reduce the resonance phenomena. They also show that a static compliance condition is necessary for high frequency cases.

1.2 Goals of current work

No studies focusing on structural vibration distribution optimization were found. Also no studies that use the Euler–Bernoulli beam with the thickness distribution as design variable were found. The goals for the current work are:

- obtain qualitative and quantitative relations between the frequency response of the Euler–Bernoulli beam and its thickness distribution;
- determine if the Euler–Bernoulli beam thickness optimization is useful for the reduction of vibrations in specific part of beam that is of relevance to feedback from the loudspeaker to the microphone in a conference phone.

2 Modelling concepts

2.1 Conference phones and the Euler–Bernoulli beam model

Structural coupling between the loudspeaker and the microphone can happen through the top or the bottom part of the casing. In both cases, the casing region between the loudspeaker and the microphone can be identified and used for simulations (see Fig. 2.1). A thin plate with characteristic dimensions L_x , L_y and L_z (see Fig. 2.2) is selected as the simplest possible model of the conference phone casing fragment. In order to avoid confusion with elastic plate models, the thin plate is later denoted as beam.

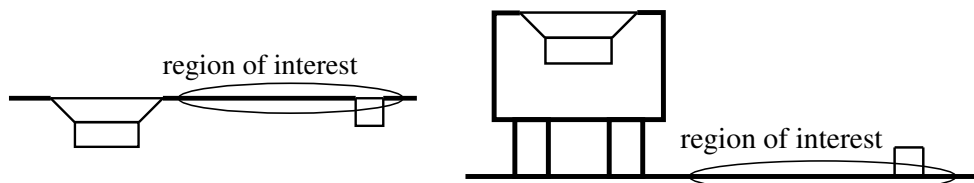


Figure 2.1: A sketch of two possible loudspeaker and microphone coupling regions.



Figure 2.2: A sketch of the considered test domain.

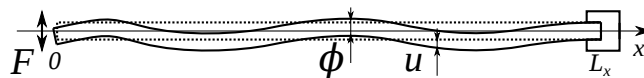


Figure 2.3: Sketch of clamped beam subjected to time harmonic loading force F , vibration amplitude not to scale.

The beam is driven with a time harmonic shear force $F = F_0 e^{-i\omega t}$ at the left side and is clamped (no linear or rotational motion) at the right side (see Fig. 2.3). It is assumed that the most significant way of sound propagation is through transverse oscillations in the plate. The unloaded, dynamic Euler–Bernoulli beam equation [10] is used for the description of the plate response to the driving force, which is

$$\frac{\partial^2}{\partial x^2} \left(EI \frac{\partial^2 u}{\partial x^2} \right) = -\mu \frac{\partial^2 u}{\partial t^2}, \quad (2.1)$$

where E is the Young modulus of material, I is the inertial momentum of the beam cross section, and μ is the linear mass density.

From now on, the local thickness is set as a variable ϕ instead of the constant L_y . Assuming time harmonic oscillations of the beam deflection $u(x, t) = \tilde{u}(x)e^{-i\omega t}$, using the explicit expression for the inertial momentum of a rectangle $I = L_z\phi^3(x)/12$, and using the explicit expression for linear mass density $\mu = m/L_x = \rho L_z\phi$ give the following Euler–Bernoulli equation for deflection amplitudes

$$\frac{d^2}{dx^2} \left(E \frac{L_z\phi^3(x)}{12} \frac{d^2\tilde{u}}{dx^2} \right) = \rho L_z\phi(x)\omega^2\tilde{u}, \quad (2.2)$$

where ρ is the density of the beam material. The equation can be rewritten in the dimensionless form (omitting tilde signs for convenience)

$$(\phi^3 u'')'' = \omega^2 \phi u, \quad (2.3)$$

using the parameter scales

$$L_x, L_y, L_z, u_0, \rho, E, \quad (2.4)$$

$$\omega_0 = \frac{L_y}{L_x^2} \sqrt{\frac{E}{12\rho}}, \quad (2.5)$$

$$F_0 = \frac{EL_z L_y^3 u_0}{12L_x^3}, \quad (2.6)$$

where L_x , L_y and L_z are the original beam dimensions, u_0 is the deflection scale, ρ and E are the beam material parameters, ω_0 is the frequency scale, and F_0 is the force scale.

Two boundary conditions for the right side of the beam are considered. The problem formulation with the clamped and the damped right boundary condition for deflection amplitudes in dimensionless formulation is

$$\begin{aligned} (\phi^3 u'')'' &= \omega^2 \phi u, \\ \text{driving force (left side): } \phi^3 u''|_0 &= 0, \quad (\phi^3 u'')'|_0 = F, \\ \text{clamped beam (right side): } u|_1 &= 0, \quad u'|_1 = 0, \\ \text{damped beam (right side): } \phi^3 u''|_1 &= 0, \quad (\phi^3 u'')'|_1 = -i\omega u \cdot \xi, \end{aligned} \quad (2.7)$$

where ξ is a damping coefficient for the right beam boundary (empirically set to $\xi = 10$ kg/s). The formulated problem is solved with MATLAB scripts using FEM with a piecewise cubic spline basis for the deflection u . The beam thickness ϕ is approximated using a piecewise constant function. The variational formulation for the clamped beam case is to find the deflection $u \in V_0$ such that

$$-v(0)F + \int_0^1 (\phi^3 u'') v'' dx = \omega^2 \int_0^1 \phi u v dx \quad \forall v \in V_0, \quad (2.8)$$

where the definition of function space V_0 includes the clamped boundary condition.

2.2 Parameters for the Euler–Bernoulli beam

The dimensions used for simulations are deduced empirically from observations of the conference phone in the UMIT laboratory seminar room to be

$$\begin{aligned} L_x &= 0.280 \text{ m}, \\ L_z &= 0.070 \text{ m}, \\ L_y &= 0.005 \text{ m}, \\ u_0 &= 1.0 \mu\text{m}. \end{aligned} \tag{2.9}$$

where the deflection scale u_0 is selected to be very small for solid material sound waves. The scale u_0 is needed if one wants to use a dimensional driving force with scale (2.6) for the simulations. Acrylonitrile Butadiene Styrene (ABS) plastic is selected as the material for the beam. The material properties according to eFunda [5] are

$$\begin{aligned} \rho &\approx 1.1 \text{ g/cm}^3, \\ E &\approx 360 \text{ MPa}. \end{aligned} \tag{2.10}$$

Other scales can be derived according to respective expressions, such as (2.5) and (2.6). For the simulations with COMSOL, the Poissons ratio $\nu = 0.35$ from Engineers Edge [4] is also used.

2.3 Optimization formulation

A measure (objective function) is introduced to describe quantitatively the feedback from the loudspeaker to the microphone. It is assumed that microphone is located from m_a to m_b on the beam (see Fig. 2.4).

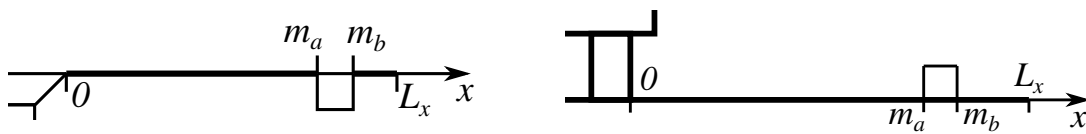


Figure 2.4: A sketch of two microphone location interpretations.

The microphone region $x \in [m_a; m_b]$ vibrates with an amplitude $u(x)$. A quantity that is proportional to the average squared vibration amplitude is selected as objective function,

$$J(\phi, \omega) = \frac{1}{2} \int_{m_a}^{m_b} |u|^2 dx. \tag{2.11}$$

The dimensional form of the objective function for the Euler–Bernoulli beam model measurements is

$$J_{\text{ID}} = \frac{1}{2L_x} \int_{m_a}^{m_b} |u|^2 dx \quad [\mu\text{m}^2], \tag{2.12}$$

and corresponding measure for 3D simulation case is

$$J_{3D} = \frac{1}{2L_x} \int_{m_a}^{m_b} \left(\frac{1}{L_z} \int_0^{L_z} |u|^2 dz \right) dx = \frac{1}{2L_x L_z} \int_{\partial M} |u|^2 dS \quad [\mu\text{m}^2], \quad (2.13)$$

which is used for frequency response calculations in 3D.

The goal is to reduce the vibrations in the microphone region as much as possible, while keeping the shape of the beam reasonable. Minimal and maximal thickness constraints, beam weight constraint, and static load compliance constraint are added to the formulation. The optimization formulation for the clamped beam driven with the force F at the frequency ω is set as

$$\begin{aligned} \min_{\phi \in L^\infty(0,1)} J(\phi, \omega) \quad \text{subject to} \\ (\phi^3 u'')'' = \omega^2 \phi u \quad \text{in } (0, 1), \\ \phi^3 u''|_0 = 0, \quad (\phi^3 u'')'|_0 = F, \\ u|_1 = 0, \quad u'|_1 = 0, \\ \int_0^1 \phi dx \leq \gamma_w, \quad \int_0^1 u_{\text{stat}} dx \leq \gamma_c C_{\text{ref}}, \\ \underline{\phi} \leq \phi \leq \bar{\phi}, \\ \phi = \phi_{\text{mic}} = \text{const} \quad \text{in } (m_a, m_b), \end{aligned} \quad (2.14)$$

where γ_w is the weight constraint, u_{stat} is the static deflection of the beam, γ_c is static compliance constraint coefficient (with respect to a reference compliance C_{ref}), and $\underline{\phi}$ and $\bar{\phi}$ is the lower and the upper limit of beam thickness, respectively. The optimization formulation for the damped beam case is the same, except for the boundary conditions at the right side of the beam. The static compliance u_{stat} is obtained solving corresponding static problem of loaded, simply supported beam

$$\begin{aligned} (\phi^3 u''_{\text{stat}})'' = 1 \quad \text{in } (0, 1), \\ u_{\text{stat}}|_0 = \phi^3 u''_{\text{stat}}|_0 = 0, \\ u_{\text{stat}}|_1 = \phi^3 u''_{\text{stat}}|_1 = 0. \end{aligned} \quad (2.15)$$

Multiple frequencies ω_i are selected for the optimization from the vocal frequency (VF) band

$$\omega = 2\pi (300, 3400) \text{ rad/s}. \quad (2.16)$$

Six frequency windows with width 500 Hz are also selected for the optimization, they are

$$\omega_{\text{FW}_n} \in 2\pi (300 + 500n, 800 + 500n) \text{ rad/s}, \quad (2.17)$$

where $n = 0, \dots, 5$ is the frequency window index. In each simulation, the frequency response is measured in a frequency interval

$$\omega_{\text{disp}} \in 2\pi (100, 4000) \text{ rad/s.} \quad (2.18)$$

The optimization problem is solved using *fmincon* from MATLAB Optimization Toolbox. Gradients of the objective function and of the compliance constraint are supplied by solving associated adjoint equations. For illustration, the adjoint equation for the objective function in clamped beam case reads: with given deflection amplitudes u , find $z \in V_0$ such that

$$-\omega^2 \int_0^1 \phi \delta u z \, dx + \int_0^1 (\phi^3 \delta u'') z'' \, dx = \int_I u \delta u \, dx. \quad \forall \delta u \in V_0 \quad (2.19)$$

The directional derivative of the objective function (2.11) with given deflection amplitudes u and given adjoint equation solution z is

$$\delta J(\phi, \omega, \delta \phi) = \int_I u \delta u \, dx = \int_0^1 [\omega^2 u z - 3\phi^2 u'' z''] \delta \phi \, dx. \quad (2.20)$$

3 Initial Euler–Bernoulli beam model verification

3.1 Eigenmode study

To verify the Euler–Bernoulli beam model, a few 3D solid mechanics simulations with COMSOL are carried out. As first, the eigenmode problem for a uniform beam is considered. The five lowest transverse eigenmode (denoted as “TMode”) frequencies obtained from the Euler–Bernoulli model f_{EB} and the 3D solid mechanics description f_{SM} are shown in Tab. 3.1. A few eigenmode shapes are also investigated; the fourth TMode deflection (see left part of Fig. 3.1) and the fourth TMode 3D shape (see right part of Fig. 3.1) are given for illustration.

TMode	f_{EB}	f_{SM}	Error
1	5.894 Hz	6.034 Hz	2.3%
2	36.94 Hz	37.70 Hz	2.0%
3	103.4 Hz	105.8 Hz	2.3%
4	202.7 Hz	207.8 Hz	2.5%
5	335.0 Hz	344.0 Hz	2.6%

Table 3.1: Eigenmode frequencies of the uniform beam, the Euler–Bernoulli model f_{EB} and the solid mechanics description f_{SM} .

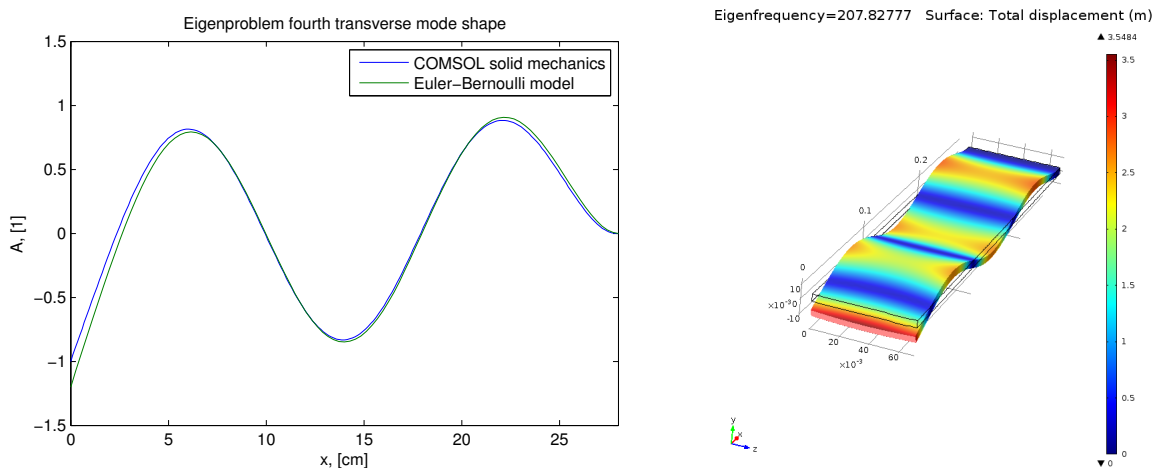


Figure 3.1: The fourth TMode eigenmode deflection for the 3D solid mechanics simulation and the Euler–Bernoulli model simulation (left), 3D shape of the solid state simulation (right).

3.2 Frequency response study

To further verify the Euler–Bernoulli beam model, the frequency response measurements in the interval $f_{\text{disp}} = (100, 4000)$ Hz for both the Euler–Bernoulli model and the 3D solid mechanics description are carried out (see Fig. 3.2). Overall, the agreement is good. The excess peaks for 3D solid mechanics description are oscillation modes (such as the one shown in Fig. 3.3), that are not modelled by the Euler–Bernoulli beam equation.

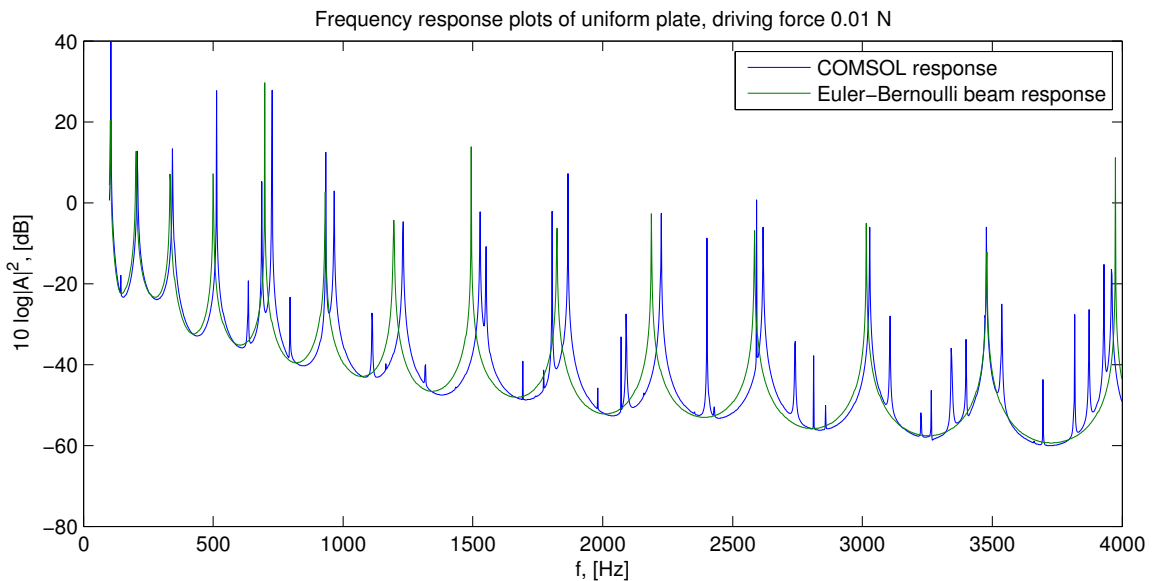


Figure 3.2: Frequency response measurements, the Euler–Bernoulli model and the 3D solid mechanics description for the beam with uniform thickness.

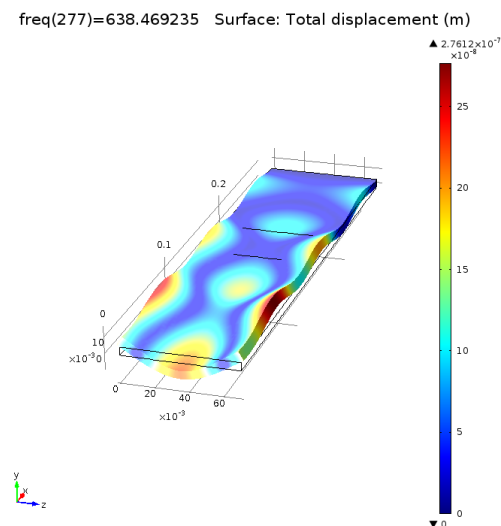


Figure 3.3: A 3D solid mechanics description response that is not modelled by the Euler–Bernoulli beam equation.

4 Results

4.1 Clamped beam

4.1.1 Broad band optimization

As first case study, the whole band $f_{\text{BB}} = (300, 3400)$ Hz is selected for optimization. Fifty equally spaced frequencies from the interval f_{BB} are selected for the objective function. An example result of the optimization procedures is shown in Fig. 4.1. Other constraint configurations were tested, similar results were obtained.

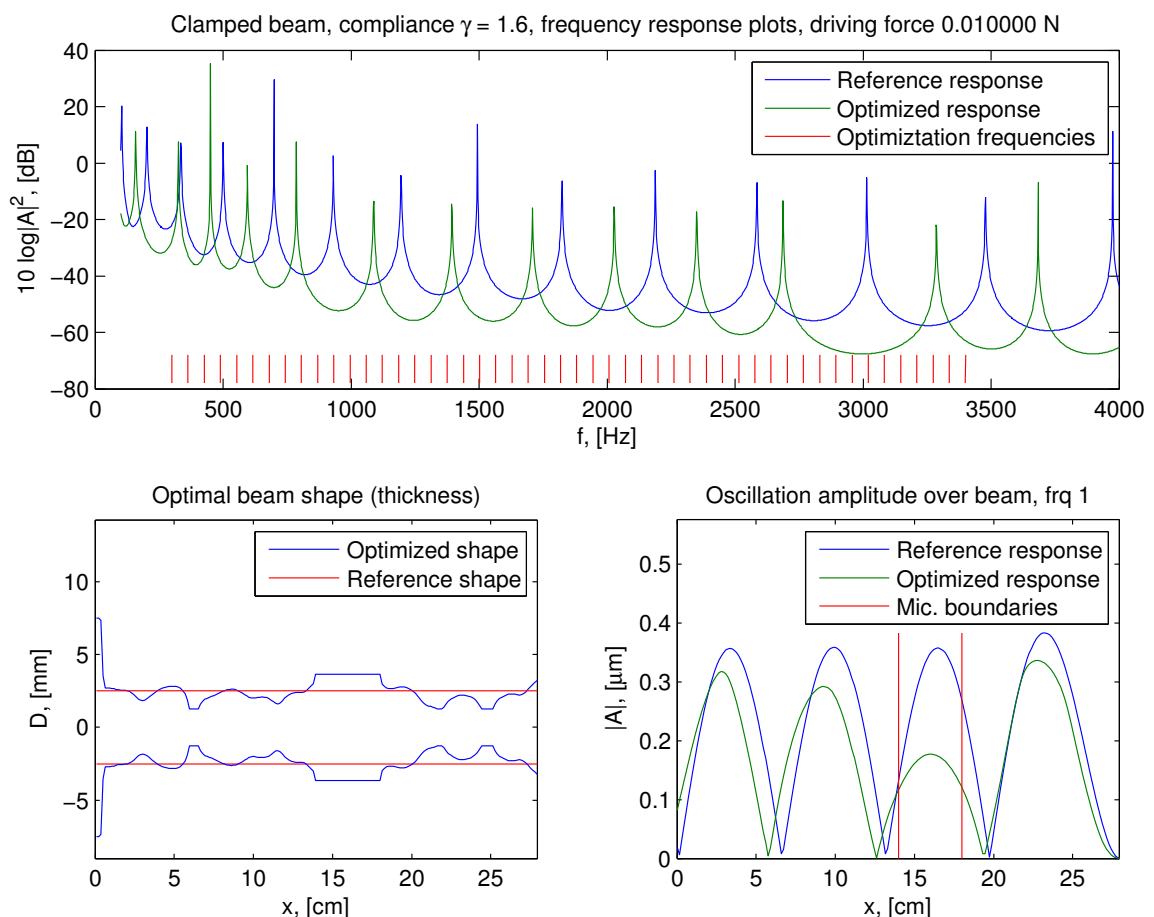


Figure 4.1: An example broad band optimization result for the clamped beam. The frequency response, the optimal beam shape, and the vibration response over the beam for the first optimization frequency is shown.

4.1.2 Pass band optimization

In further tests, six 500 Hz frequency windows $f_{FWn} = (300 + 500n, 800 + 500n)$ Hz with $n = 0, \dots, 5$ are considered. In each window, fifty equally spaced frequencies are selected for the objective function. Example results of low and high frequency window optimizations are shown in Fig. 4.2 and Fig. 4.3, respectively. In this case, filtering is applied to the design variable to smooth the shape of the design. The filtering is of a similar kind that is routinely applied in topology optimization of elastic structures [1, pages 35 – 36]. The best performing beam shape (see Fig. 4.3) is imported into COMSOL simulation software for verification. The beam shape imported in COMSOL software at correct proportions is shown in Fig. 4.5. The frequency response plots from COMSOL simulations are shown in Fig. 4.4.

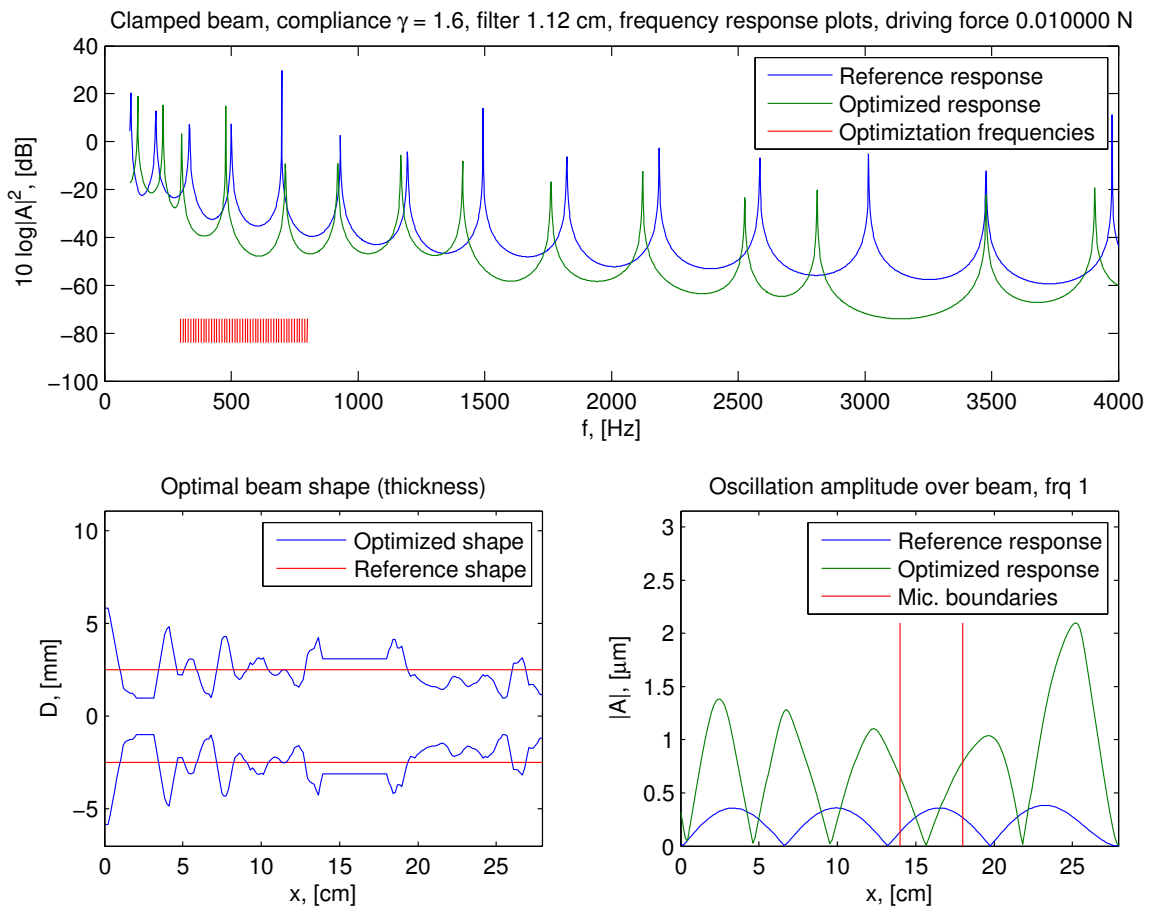


Figure 4.2: An example frequency window (300, 800) Hz optimization result for clamped beam. The frequency response, the optimal beam shape, and the vibration response over the beam for the first optimization frequency is shown.

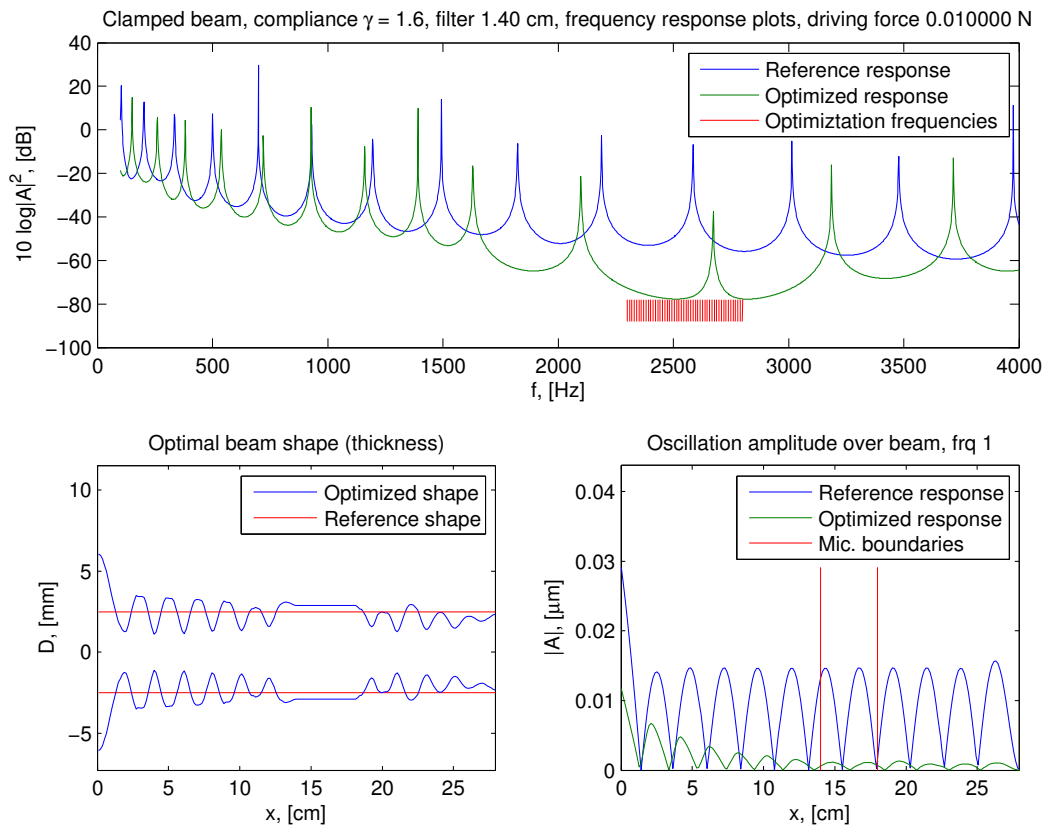


Figure 4.3: An example frequency window (2300, 2800) Hz optimization result for clamped beam. The frequency response, the optimal beam shape, and the vibration response over the beam for the first optimization frequency is shown.

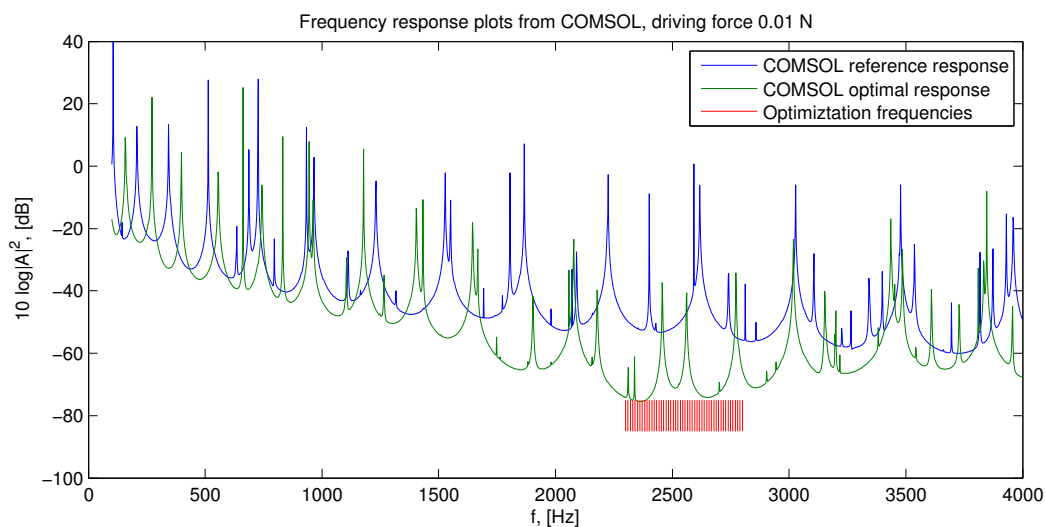


Figure 4.4: A verification of the optimal beam shape for frequency window 4 (see Fig. 4.3). The frequency response results from the COMSOL solid mechanics simulations are shown.

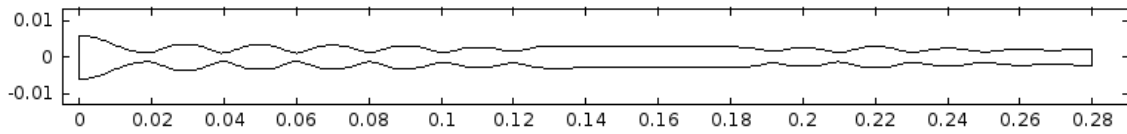


Figure 4.5: The optimal beam shape (see Fig. 4.3) in correct proportions, imported in COMSOL software.

4.2 Damped beam

4.2.1 Broad band optimization

For the damped beam, the first case study also concerns the broad band frequency interval f_{BB} . The same fifty frequencies as in Section 4.1.1 are selected for the objective function. An example result of optimization is shown in Fig. 4.6. Other constraint configurations were tested, similar results were obtained.

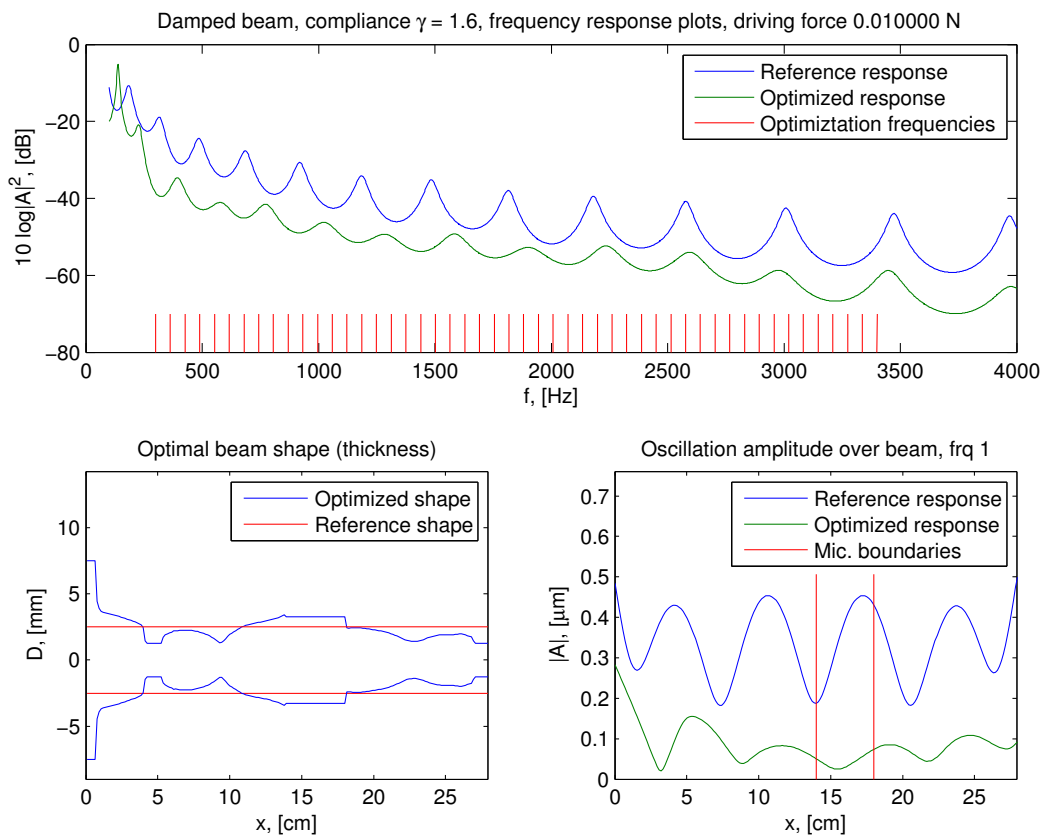


Figure 4.6: An example broad band optimization result for the damped beam. The frequency response, the optimal beam shape, and the vibration response over the beam for the first optimization frequency is shown.

4.2.2 Pass band optimization

For the damped beam, frequency window f_{FWn} optimizations were also done. An example result of a high frequency window optimization is shown in Fig. 4.7. For lower frequencies, the performance was smaller compared to the higher frequencies. The same type of behaviour was already observed in the clamped beam case (compare Fig. 4.2 and Fig. 4.3). The overall performance is much better in the damped beam case. No 3D verification has been done for the damped beam case.

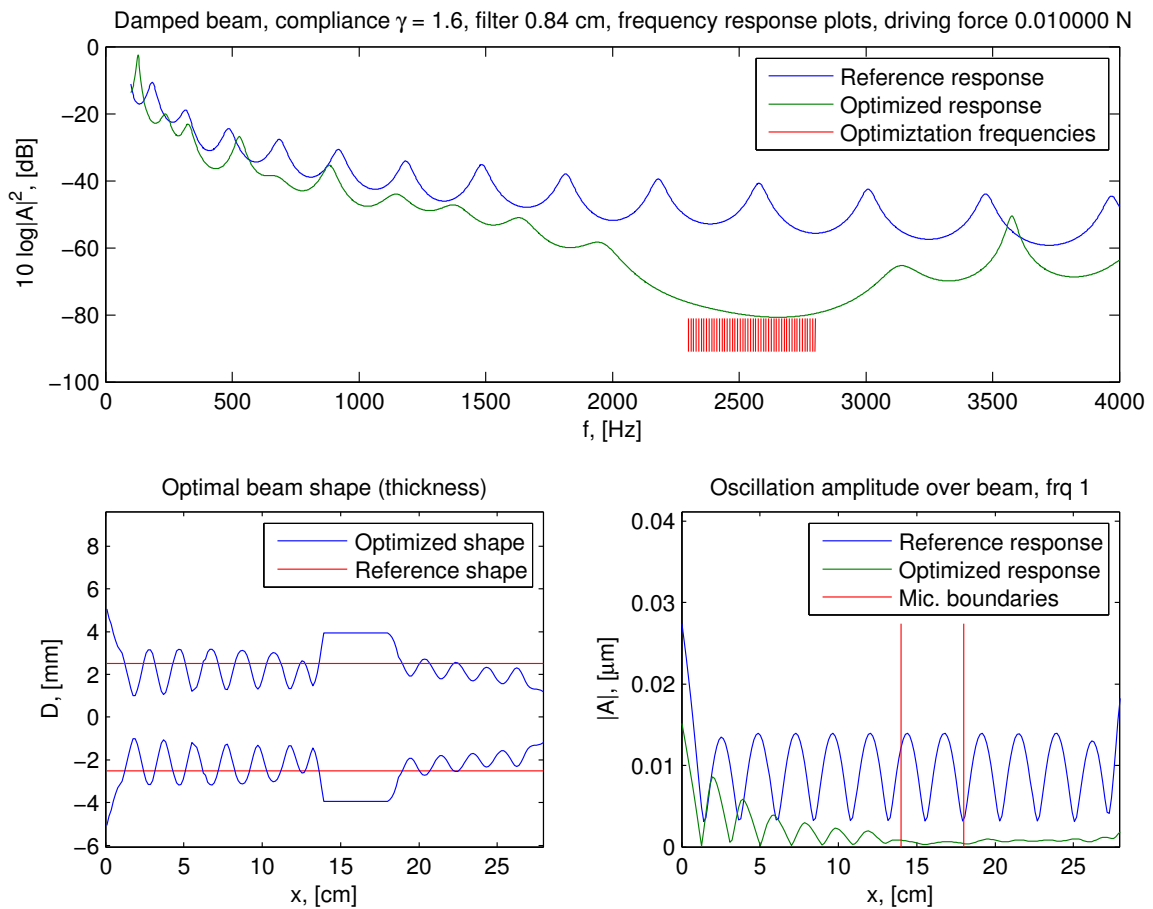


Figure 4.7: An example frequency window (2300, 2800) Hz optimization result for damped beam. The frequency response, the optimal beam shape, and the vibration response over the beam for the first optimization frequency is shown.

5 Conclusions and further work

The results suggest that the optimization with the Euler–Bernoulli beam model can give interesting results that are of relevance to conference phone casing constructions. Selected simulations has been verified using the full 3D solid mechanics description in COMSOL. Conclusions from the optimization studies:

- optimization using frequency windows gives better performance than broad band optimization;
- optimization at higher frequencies gives better performance than optimization at lower frequencies;
- optimization using the damped beam description gives better performance than using the clamped beam description.

After investigations of a large number of optimization results, following mechanisms were consistently observed in the optimized shapes:

- In broad band cases, a thickening occurs at the driving force side and also in the microphone region.
- In pass band cases, a band-gap phenomenon occurs. The optimization procedure produces quasi-periodic shapes with a wave length similar to the wave length of the forced vibrations. The structure is similar to those occurring in phononic band gap materials [1, pages 138 – 148].

Further investigations with topology optimization of 2D and 3D elasticity or shell models can be of interest. Olhoff et al. [7] have carried out topology optimization studies for structures subjected to a forced vibration. However, their work is not directly related to the design of phone casings.

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