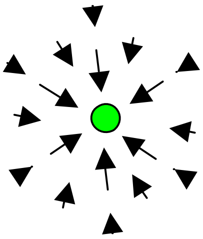
Robot Navigation with Potential Fields



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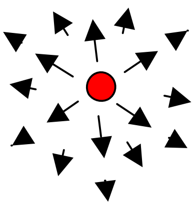

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1. Introduction

One important task in robot navigation is Path Planning: finding a viable way between a starting and a goal point. Several methods have been developed, typically divided into deliberative and reactive approaches. In the deliberative approach, a path through a map of the world is constructed, followed by the execution of the plan by the robot. In the reactive approach, the robot's sensors and actuators directly generate the navigation behavior. Deliberative controllers require high computational power and have slow response times. Reactive controllers respond fast but suffer from the restrictions of not having a global picture of the environment and the path planning task.

In this compendium we will describe navigation based on *Potential Fields*, a common reactive approach for path planning, originally proposed by Khatib (1985, 1990). The basic idea is to guide the robot by defining attractive and repulsive forces representing goal and obstacles respectively. A clear analogy can be made to animals trying to reach goal locations and avoiding obstacles.

Figure 1. The fish act as if repelled by a uniform force emanating from the shark.

Figure 2. The sheep act as if attracted by a uniform force emanating from the man.

In Figure 1, the fish keeps away from the shark as if repelled by a uniform force field surrounding the shark. In Figure 2, the sheep are attracted by the man (or more likely by the food he brought) in a similar fashion. Arbib and House (1987) describe experiments with toads trying to reach some worms placed behind an obstacle fence. The observed motion of the toad is illustrated by the arrows in Figure 3. The arrows indicate direction and speed of the toad when being placed at different locations. The motion can be described as a combination of attraction to the location of the worms and repulsion to the location of the fence. The potential field method attempts to formalize this kind of navigation strategy.

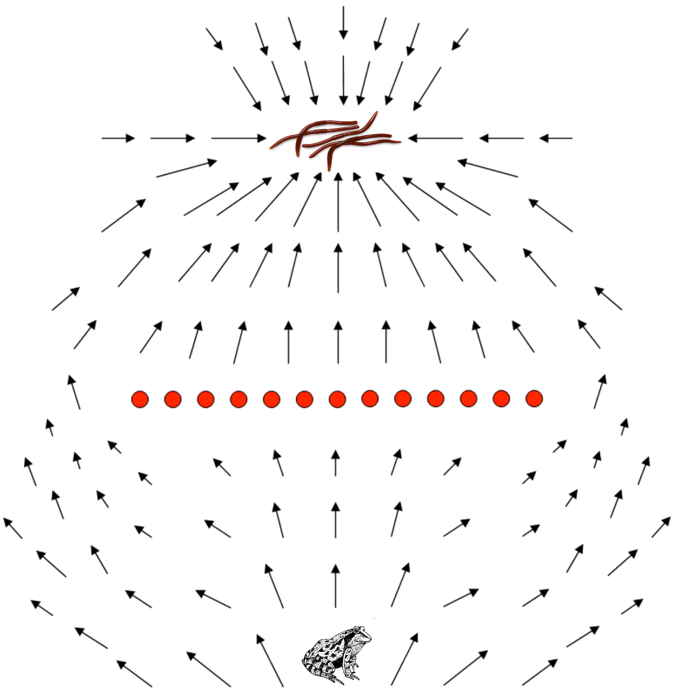

Figure 3. Observed direction and speed of motion of a toad, trying to reach the worms placed behind the red fence.

2. Potential Fields for Robot Navigation

Switching focus to robotics, suppose we want to construct a robot that can navigate over a field occupied by an obstacle such as a steep rock, until it reaches a goal position G at the other end of the field. For the robot position **q** we define two potential field functions U_{rep} and U_{att} such that U_{att} converges toward zero for **q** close to G, and U_{rep} is zero if **q** is well off the obstacle and gradually takes larger values as **q** gets closer to the obstacle. The following definitions satisfy these requirements:

$$U_{rep}\text{: 1/(distance to obstacle)} \quad (1)$$
$$U_{att}\text{: distance to G} \quad (2)$$

We also define associated force vectors acting on the robot, as the negative gradients of the potential fields:

$$\mathbf{F}_{rep} = -\nabla U_{rep} \quad (3)$$
$$\mathbf{F}_{att} = -\nabla U_{att}. \quad (4)$$

Consequently, the vector $\mathbf{F}_{rep}$ points from the robot away from the obstacle, and $\mathbf{F}_{att}$ points from the robot toward G. A strategy to move safely toward G is to move in the direction of a vector **F**:

$$\mathbf{F} = \mathbf{F}_{rep} + \mathbf{F}_{att}. \quad (5)$$

By moving along **F**, the robot will move toward the goal while staying away from the obstacle.

The method can be easily extended to include several obstacles. As we will see, several other types of potential fields, with associated force fields, can also be defined. The general expression for **F** computed at a point (x,y) is:

$$\mathbf{F}(x,y) = \sum_i \mathbf{F}_i(x,y) \tag{6}$$

where $\mathbf{F}_i(x,y)$ is the force vector associated with potential field U_i, i.e. the negative gradient of U_i :

$$\mathbf{F}_\mathrm{i}\ (x,y) = -\,\nabla \mathrm{U}_\mathrm{i}(x,y). \tag{7}$$

An algorithm for robot navigation from a position (x,y) using the potential fields methods comprises the following three steps that are repeated until the goal is reached:

1. Compute force vectors $\mathbf{F}_i\ (x,y)$ for all potential fields
2. Combine all force vectors into $\mathbf{F}(x,y)$ using Equation 6.
3. Move along $\mathbf{F}(x,y)$ with speed proportional to $|\mathbf{F}(x,y)|$.

Force vectors are usually computed by sensors estimating distances and directions to objects surrounding the robot. These values are then input in the equations defining the forces. Note that force vectors only have to be computed for current position (x,y). Illustrations of the entire force fields are valuable for describing and understanding the method, but the robot never computes this vector field, just the local vectors $\mathbf{F}_i\ (x,y)$. Also note that potential field values U_{rep} and U_{att} are normally not computed explicitly, since the robot motion in step 3 above is determined completely by **F**.

The method can be described with an analogy to navigation in a physical 3D landscape, as illustrated in Figure 4, with one attractive goal at coordinates (1, -2) and three repulsive objects. The goal can be thought of as located at the bottom of a valley, and obstacles as steep rocks. The sum U of all potential fields is shown as the z value for each point (x,y). The shown paths, from five different starting points to the goal, is the motion of an imagined ball rolling freely in the landscape. If placed on the slope of a rock, it will roll down and continue in the direction of maximum descent until it reaches the goal[1]. The trajectory goes toward the goal; along the valleys in between the obstacles.
In Figure 5, the force **F** that causes the ball to move is plotted as function of x and y. Each arrow represents the direction and strength of **F** at the location of the arrow. To make the figure more readable, the arctan of the strength of **F** is displayed. For the same reason, the magnitude of the repulsive potential fields in Figure 4 is limited to 2. The paths are drawn using a gray scale to illustrate the robot speed $|\mathbf{F}|$, with black corresponding to full speed and light gray to very low speed. The definitions of potential field functions U_{rep} and U_{att} (Equations 1 and 2), are somewhat modified as described in the following section.

[1] As always, the analogy is not complete. In this case, one missing aspect is the inertia of the ball. In reality it would cause the ball to ”over shoot” both the ideal path and the goal.

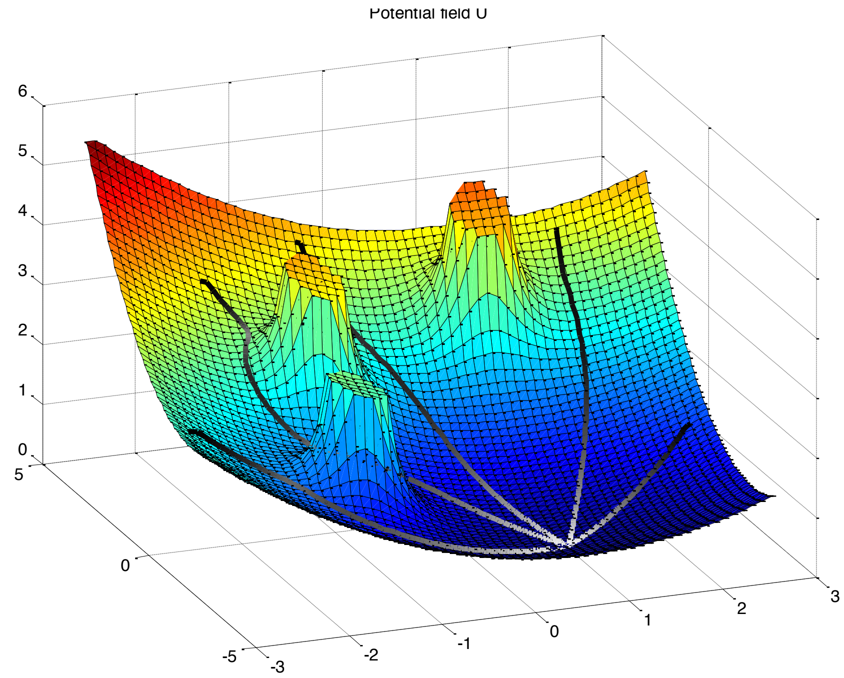

Figure 4. Example of a potential field $U(x,y)$, constructed from 3 obstacles and one goal. The attractive goal is located at (1,-2).

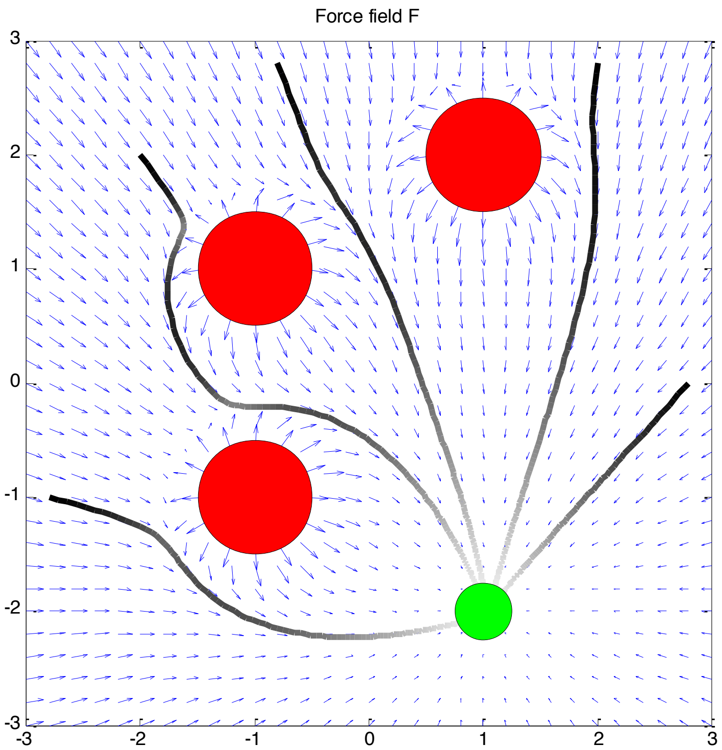

Figure 5. Force vector field **F** computed as the negative gradient of the potential field in Figure 4. Red large circles represent repulsive obstacles and the green small circle represents the attractive goal. Paths following the force vectors, from five different starting positions are also shown.

3. Alternative Shapes of Potential Fields

In the first example, the potential fields U_{rep} and U_{att} are defined in a simplified manner (Equations 1 and 2). Khatib (1985, 1990) suggests the following definition for an attractive field centered at the point $\mathbf{q}_a$:

$$U_{att} = \xi d^2 / 2 \tag{8}$$

where $d = |\mathbf{q} - \mathbf{q}_a|$; $\mathbf{q}$ is the current position of the robot and ξ is an adjustable constant. The corresponding gradient ∇U_{att} is given by

$$\nabla U_{att} = \xi\,(\mathbf{q} - \mathbf{q}_a). \tag{9}$$

Khatib (1985, 1990) suggests the following repulsive field U_{rep} surrounding an obstacle located at a point $\mathbf{q}_o$:

$$U_{rep} = \begin{Bmatrix} \frac{1}{2}\eta\left(\frac{1}{d} - \frac{1}{d_o}\right)^2 & d \le d_o \\ 0 & d > d_o \end{Bmatrix} \tag{10}$$

where $d = |\mathbf{q} - \mathbf{q}_o|$; $\mathbf{q}$ is the current position of the robot; d_o is the influence distance of the force and η is an adjustable parameter. The corresponding gradient ∇U_{rep} is given by

$$\nabla U_{rep} = \begin{Bmatrix} \eta\left(\frac{1}{d} - \frac{1}{d_o}\right)\frac{(\mathbf{q} - \mathbf{q}_o)}{d^3} & d \le d_o \\ 0 & d > d_o \end{Bmatrix}. \tag{11}$$

Equations 8 to 11 were used to generate the potential field and the force field in Figures 4 and 5. The following parameters values were used: ξ=0.02, η=0.1 and d_o=1. In the literature, several other types of attractive and repulsive potential fields have been suggested; see for instance Castañeda et al. (2008). In the next section we will introduce a number of totally different kinds of potential fields.

4. Basic Types of Potential Fields

In addition to variants of the basic attractive and repulsive potential fields, a number of other types have been suggested. By combining them, a navigation strategy can be tailored to specific scenarios and address specific needs. Murphy (2000) describes seven common fields illustrated in Figure 6. The *Attraction* and *Repulsion* fields have, as described above, shapes that direct the robot toward or away from a defined point. Sometimes, no specific target point is defined for the navigation task, but rather a general preferred direction. One example is when the robot is moving through a corridor. In such case a *Uniform* field may be used. A repulsive force may originate not from a point but rather from an entire surface such as a wall. This is implemented by the *Perpendicular* field. The *Tangential* field may be used to direct the robot around an obstacle (sometimes in a task specific direction such as in the NaT fields described below), or to investigate an object without getting too close to it. The *Selective attraction* field behaves like an *Attraction* field, but only within a limited angular sector. A *Random* field has random direction and magnitude regardless of position. This kind of field is sometimes used to overcome the problems with local minima (see below).

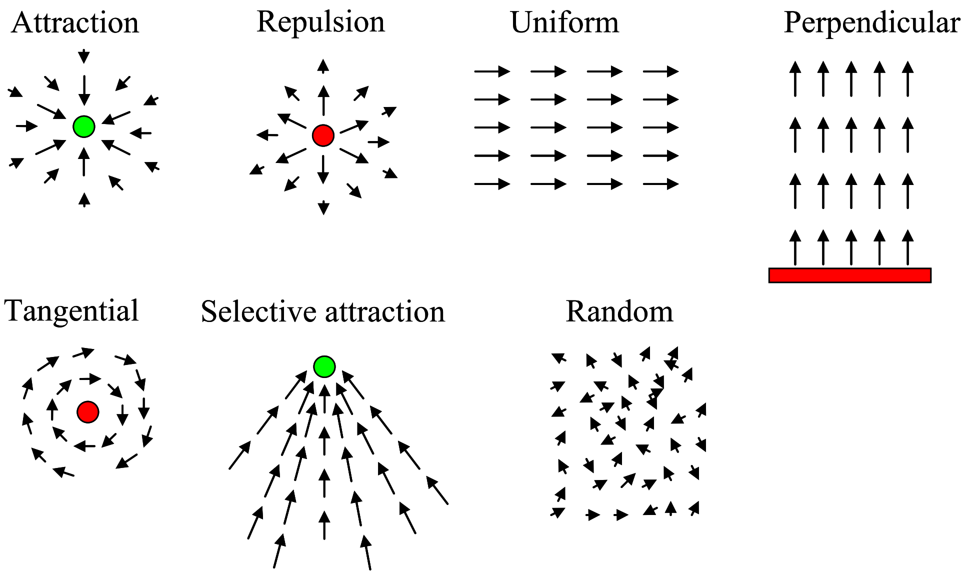

Figure 6. Seven basic types of potential fields (or more correctly: the associated force fields). By combining them, complex robot behaviors can be achieved.

Note that this kind of figures with arrows show the force field **F**, i.e. the gradient of the potential field function. Each arrow indicates strength and direction of the force the robot will "feel" when navigating. Since this force is what matters in most cases, the force field is often, somewhat sloppily, denoted potential field, while the actual potential field function U is not illustrated or even defined explicitly.

5. Combining Several Potential Fields

In the example given in Section 2, a basic "move toward goal without colliding" behavior is achieved by interacting potential fields. The force fields $\mathbf{F}_i$ associated with each potential field are combined by simple addition (Equation 6). The method may also be used to produce more complex behaviors, and also sequencing of sub-behaviors. The switch between sub-behaviors is automatically managed by interaction between the robot and the environment. In the example illustrated in Figure 7, a *Selective attraction* field is combined with one *Attraction* field and one *Tangential* field, resulting in a docking behavior. Figure 7 shows the sum of all force fields. The resulting behavior will be that the robot approaches the target from any position, rotates around it until the *Selective Attraction* field causes it to proceed to the goal position, for instance for docking with a battery charger.

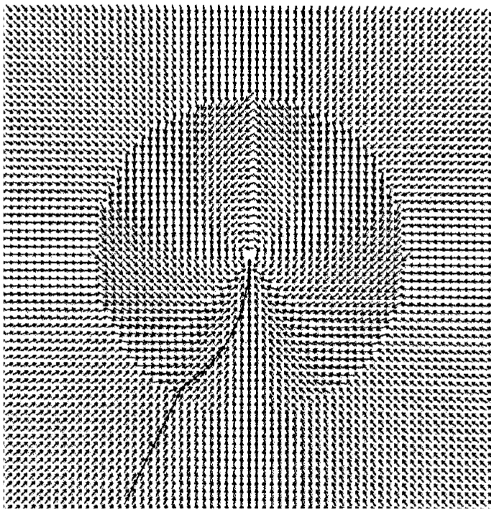

Figure 7. One *Selective attraction* field combined with one *Attraction* and one *Tangential* field cause the robot to approach the goal, swirl around it and finally dock from a pre-determined direction (from Murphy 2000).

In this way, sequencing of sub-behaviors is achieved by interaction between the environment and the potential fields. Information from the sensors decides when and how the swirling and final docking should take place.

While the result can be seen as several independent and interacting behaviors, it can also be seen as one single potential field: Sensor data is used to compute one force vector which then controls the robot. This suggests a recursive approach, in which several behaviors $\mathbf{B}_i$, each one implemented as a sum of other behaviors, are summed together to produce a complex overall behavior **B**. The summation can be extended with weights:

$$B = \sum_i w_i B_i . \qquad (12)$$

The weights w_i used in the summation can be either pre-determined or set by other mechanisms operating in real-time in the robot. In this way, the relative importance of behaviors can be easily adapted and even learned by the robot. A behavior can also be easily deactivated by setting the corresponding weight to zero.

Equation 12 may be illustrated graphically as exemplified in Figure 8.

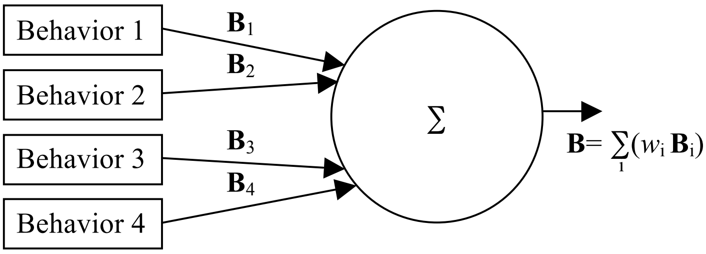

Figure 8. Coordination of four behaviors by weighted summation of force vectors.

In the figure, four behaviors are coordinated; each one represented by a potential field, or recursively by other coordinated behaviors. The output force vectors $\mathbf{B}_1,\ldots,\mathbf{B}_4$ are summed and weighted to produce one compound vector output **B** that controls the robot or serves as input to other coordinated behaviors.

6. Local Minima

The basic principle with potential field navigation is that the vector field *F* gradually approaches zero close to the goal position, such that the robot will slow down and eventually stop at the goal. A common problem is the presence of multiple, local minima (Borenstein and Koren 1991), (Grefenstette and Schultz 1994). More precisely, that vector field *F* often contains several local minima, such that the robot may get stuck in positions far away from the defined goal position. The phenomenon is illustrated with one attractive and one repulsive field in Figure 9. Just in front of (above in the picture) the red obstacle, the repulsive force from the obstacle will exactly counterbalance the attractive force from the goal. If the robot reaches this point, for instance when starting at position (0, 2), the resulting force will be zero, and the robot will stop. The robot may also be dragged into a local minimum, as illustrated in Figure 10. Forces from two obstacles and the goal cancel each other out in one point, and as can be seen from the plotted paths, the robot is attracted to this point from many different starting positions. Several ways to overcome the problems with local minima have been suggested. Below we will describe approaches that can be implemented within the potential field framework.

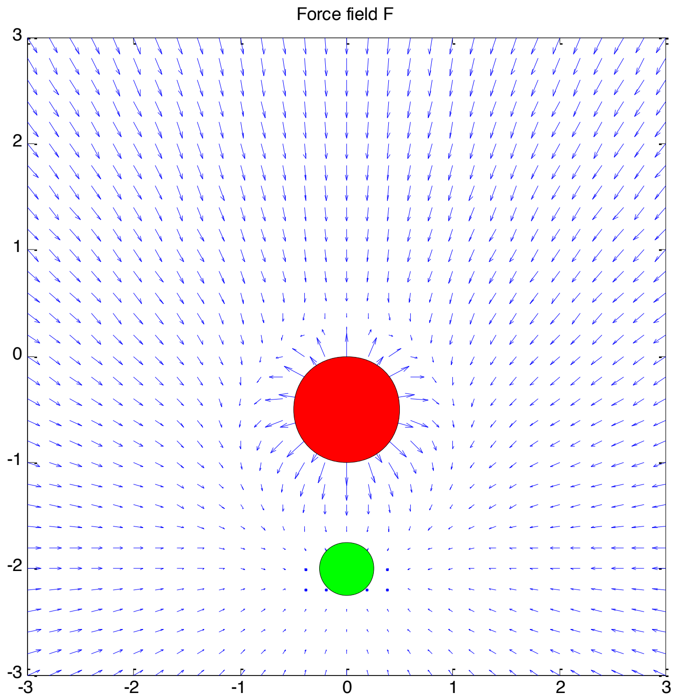

Figure 9. Example of the problem with local minima. At one point straight above the red obstacle, the repulsive force from the obstacle cancels out the attractive force from the goal. If the robot reaches this point, it will stop.

6.1 Injection of Random Noise

One approach is to using a random potential field, in which all points are assigned random values (Balch and Arkin 1993). The resulting force field has random directions and magnitudes at all points, and is simply added to the other force fields. The expected effect is that the robot "bumps" out of the local minimum, as illustrated in Figure 11. The noise field thus acts as a “reactive grease” (Arkin 1989).

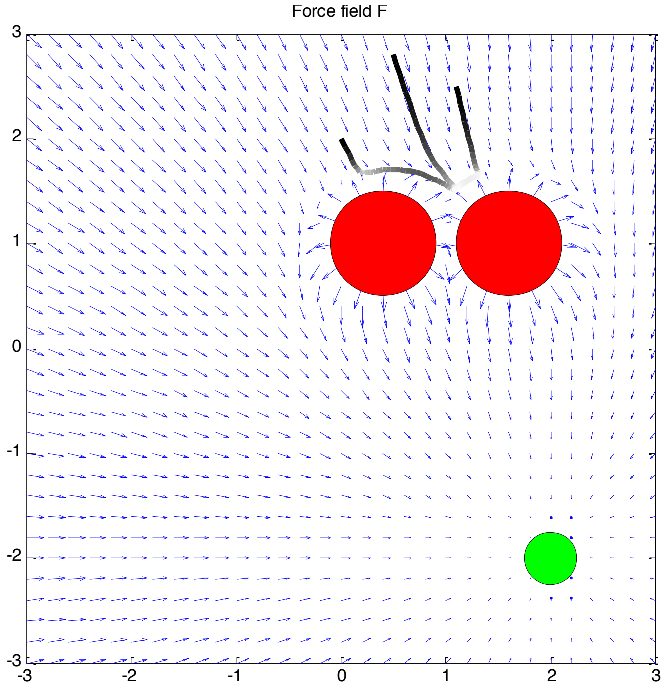

Figure 10. The robot may be dragged into a local minimum from several different starting positions. The gray scale of robot paths corresponds to the speed of the robot.

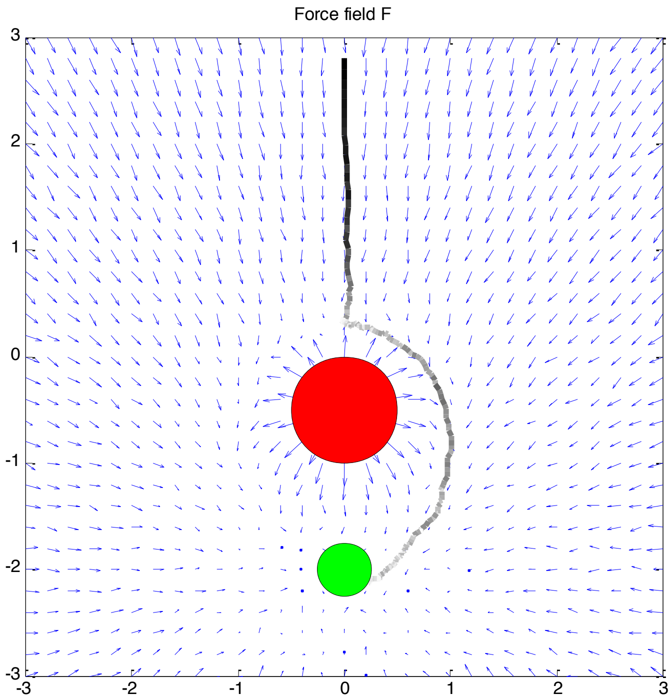

Figure 11. The local minimum in Figure 9 is avoided by adding a potential field comprising random values. The jagged path is a consequence of the random force vectors that are added at every point along the path.

6.2 Avoid-Past Behavior

Sometimes, such as in the example in Figure 10, adding noise will not solve the problem but rather introduce a cyclic behavior where the robot keeps returning to the local minimum despite minor excursions around that point. A way to overcome this is to let the robot remember where it has been and push itself away from these locations. The strategy may be implemented as a dynamically updated potential field with repulsive forces from all previously visited locations (Balch and Arkin 1993). The idea with such an *Avoid-past* behavior is illustrated in Figure 12. In the left picture, the robot runs into a local minimum and added noise does not manage to permanently guide the robot away. In the right picture, an Avoid-past behavior solves the problem and the robot manages to find its way to the goal position.

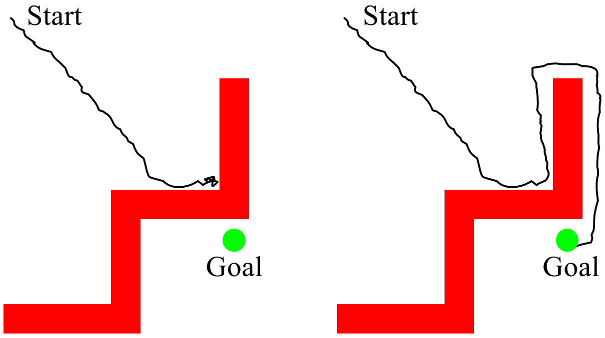

Figure 12. An Avoid-past behavior to overcome the problems with local minima and cyclic behavior.

6.3 Navigation Templates (NaT)

The regular repulsive fields around obstacles can be replaced by tangential fields (Figure 6). This reduces the risk of attractive and repulsive forces cancelling each other out in local minima. In an approach called *Navigation Templates* (Slack 1993, Gat 1993), the direction of tangential fields are set in real-time to the direction of the vector sum of all other involved potential (force) fields. In this way, obstacles will be avoided by a force vector that points in the same direction as the general behavior, normally toward the goal. This may also have other positive effects on navigation. In the example in Figure 13 (from Murphy 2000), a robot should navigate along a pier with water on both sides. This is accomplished by a uniform field directed along the pier in the desired direction, combined with two tangential fields directed perpendicular to the edges of the pier. In the left picture, a red obstacle is surrounded by a regular repulsive field. This may push the robot into the water if the robot approaches the obstacle from an unfortunate direction. Replacing the repulsive field with a navigation template avoids this by forcing the robot to avoid the obstacle in a safe direction.

Other alternative formulations of potential fields have been suggested to overcome the problems with local minima, see for instance Castañeda et al. (2008). One approach is to use harmonic functions (Kim and Khosla 1992, Connolly et al. 1993). Potential fields implemented as harmonic functions are guaranteed not to have any local minima equal to zero but are more computationally demanding.

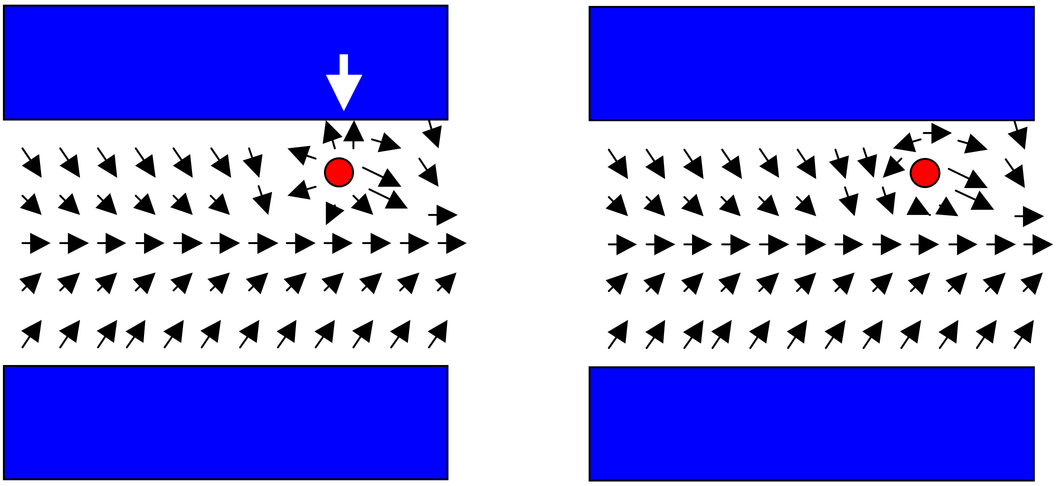

Figure 13. The regular repulsive potential field (left), may push the robot into the water (white arrow) if the robot approaches the obstacle along the upper side. A navigation template (right) avoids this by forcing the robot to avoid the obstacle in a safe direction. Modified example from (Murphy 2000).

7. Other Problems with Potential Fields

Apart from local minima and cyclic behavior, jerky motion of the robot is often mentioned as a problem with the Potential Fields approach. This is often caused by too low update rate, but may also be the result of using discontinuous potential fields with abrupt borders. Another problem is that the robot and obstacles are treated as points. This may cause collisions between the robot and obstacles, although the midpoints of the robot and the obstacles are well apart. The problem can be overcome with more elaborated shapes of the potential fields. A related problem is caused by the assumption that the robot is able to change velocity and direction instantaneously, and that the speed of the robot is not taken into account. More advanced path planning techniques take these dynamic aspects of robot motion into account.

8. Advantages with Potential Fields

Despite the problems listed above, Potential Fields have many advantages as a general technique for robot navigation.

- They are easy to implement and visualize, and the resulting behavior of the robot is therefore easy to predict by the designer.
- They support for parallelism. Each field is independent of the others and may be implemented as general software, or even hardware, modules.
- They can be easily parameterized and configured during design phase or in real-time.
- The combination mechanism (Equation 6) is flexible and can be tweaked with gains to reflect varying importance of sub-behaviors.

References

Arbib, M. A. & House, D. H. (1987), Depth and detours: an essay on visually guided behavior. In M. A. Arbib and A. R. Hanson (eds.), Vision, Brain and Cooperative Computation, 129-163, Cambridge, MA: MIT Press.

Arkin, R.C. (1989), Motor Schema-Based Mobile Robot Navigation, International Journal of Robotics Research, Vol. 8, No. 4, August 1989, pp. 92-112.

Balch, T. and Arkin, R.C. (1993), Avoiding the Past: A Simple but Effective Strategy for Reactive Navigation, Proceedings from IEEE International Conference on robotics and automation 1993.

Borenstein J. and Koren Y. (1991), The vector field histogram-fast obstacle avoidance for mobile robots, IEEE Transactions on Robotics and Automation 7, pp. 278-288.

Castañeda M.A.P., Savage J., Hernandez A. and Cosio F.A. (2008), Local Autonomous Robot Navigation using Potential Fields, In Motion Planning, (Edited by Xing-Jian Jing), InTech.

Connolly, C.I., and Grupen, R.A. (1993), The application of harmonic functions to robotics, Journal of Robotic Systems, vol. 10, October 1993, pp. 931-946.

Gat, E. (1993), Navigation templates: enhancements, extensions, and experiments, Proceedings of the IEEE International Conference on Robotics and Automation, pp. 541–547.

Grefenstette J. and Schultz A.C. (1994), An evolutionary approach to learning in robots, In Proceedings of the Machine Learning Workshop on Robot Learning, Int. Conf. on Robot Learning, pp. 65-72, New Brunswick, N J.

Khatib O. (1985), Real-time obstacle avoidance for manipulators and mobile robots, Proceedings of the IEEE International Conference on Robotics and Automation, St. Louis, MO, pp. 500-505.

Khatib O. (1990), Real-time obstacle avoidance for manipulators and mobile robots, In Autonomous Robot Vehicles, (Edited by I.J. Cox and G.T. Wilfong), pp. 396-404, Springer-Verlag.

Kim J and Khosla P. (1992), Real-Time Obstacle Avoidance Using Harmonic Potential Functions. Institute for Software Research. Paper 607. http://repository.cmu.edu/isr/607.

Murphy, R. (2000), Introduction to AI robotics, MIT Press.

Slack, M.G. (1993), Fixed computation real-time sonar fusion for local navigation, Proceedings of the 1993 IEEE International Conference on Robotics and Automation, pp.123-129.

Vaščák, J. (2007), Navigation of Mobile Robots Using Potential Fields and Computational Intelligence Means, Acta Polytechnica Hungarica, Volume 4, No.1, pp. 63-74.